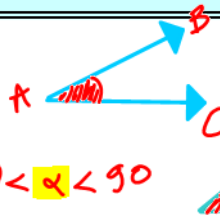
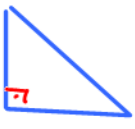
TRİGONOMETRİ

1.1 Açılar



(a) Dar Açı: $0 < \alpha < 90$

(b) Dik Açı: $\alpha = 90$



(c) Geniş Açı: $90 < \alpha < 180$

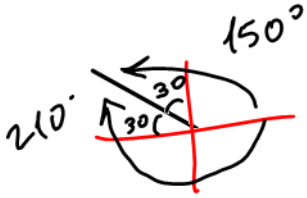
(d) Tam Açı: $\alpha = 180$

(a) Tümleyen: α, β $\quad \alpha+\beta=90$

$30°$ tümleyen $=60°$

(b) Bütünleyen: $\gamma+\theta$ $\quad \gamma+\theta=180$

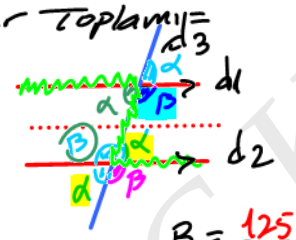
(c) konjuge:

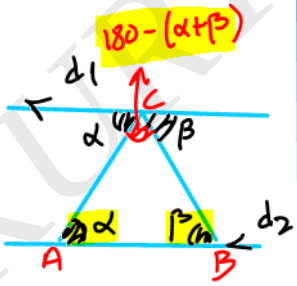


Basit bir üçgen;

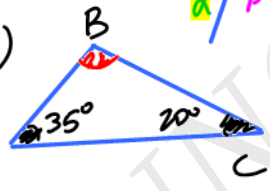
İç Açılar Toplamı $=$

$\alpha+\beta=180°$

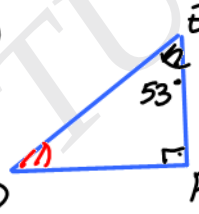




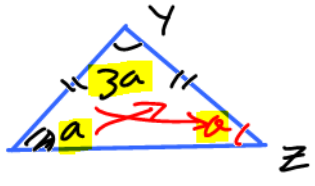
$\alpha+\beta+180-\alpha-\beta=180$

1) 

$B = 125°$

2) 

$D = 37°$

3) 

$5a = 180$

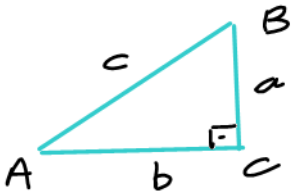
$a = 36$

$X = 36°$

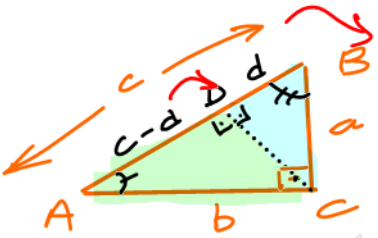
$Y = 108°$

$Z = 36°$

Pisagor Teoremi:



$a^2+b^2=c^2$



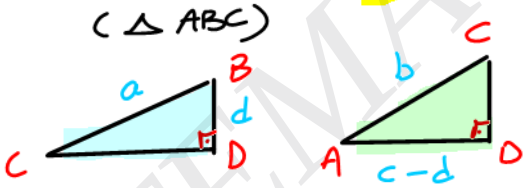
$\frac{a}{d} = \frac{c}{a}$

(*) $a^2 = cd$

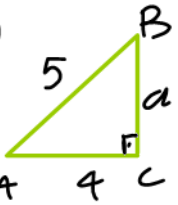
$\frac{c}{b} = \frac{b}{c-d}$

(**) $c^2 - cd = b^2$

$(\triangle ABC)$



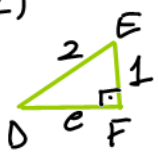
$c^2 - a^2 = b^2 \rightarrow \therefore$ $\boxed{c^2 = a^2 + b^2}$

1) 

$a = 3$

$a^2 + 4^2 = 5^2$

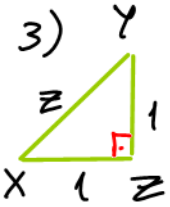
$a^2 = 25 - 16$

2) 

$e = \sqrt{3}$

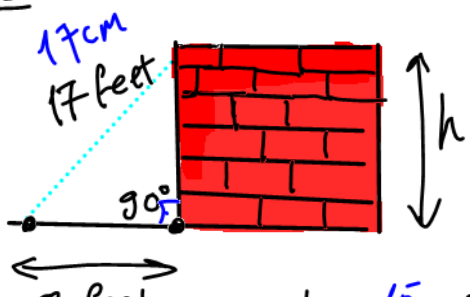
$e^2 + 1^2 = 4$

$e^2 = 3$

3) 

$z = \sqrt{2}$

Örnek:



$h = 15$ feet

$h^2 + 8^2 = 17^2$

(3, 4, 5) → (30, 40, 50)

(6, 8, 10) (8, 15, 17)

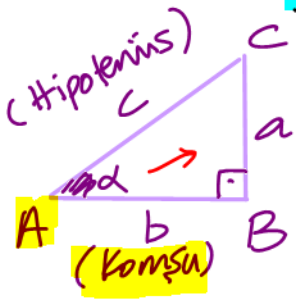
(7, 24, 25)

TUNÇKURT MATEMATİK

1.2. Dik Üçgenlerde Trigonometrik Fonksiyonlar

$\sin^2 A + \cos^2 A = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1$

$a^2+b^2=c^2$



$\sin^2 A = \frac{a^2}{c^2}$

$\cos^2 A = \frac{b^2}{c^2}$

(·) $\sin A = \frac{\text{Karşı}}{\text{Hipo}} = \frac{a}{c}$

(·) $\cos A = \frac{\text{Komşu}}{\text{Hipo}} = \frac{b}{c}$

$\frac{1}{\csc A} = \sin A$

$\tan A = \frac{\sin A}{\cos A} = \frac{\text{Karşı/Hipo}}{\text{Komşu/Hipo}} = \frac{\text{Karşı}}{\text{Komşu}}$

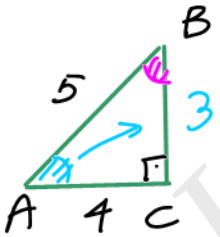
$\csc A = \frac{1}{\sin A} = \frac{\text{Hipo}}{\text{Karşı}} = \frac{c}{a}$

$\sec A = \frac{1}{\cos A} = \frac{\text{Hipo}}{\text{Komşu}} = \frac{c}{b}$

$\cot A = \frac{\cos A}{\sin A} = \frac{\text{Komşu/Hipo}}{\text{Karşı/Hipo}} = \frac{\text{Komşu}}{\text{Karşı}}$

$\cot A = \frac{1}{\tan A}$ $\quad \tan A = \frac{1}{\cot A}$

Örnek: (Dik üçgen)



$\sin A = \frac{3}{5}$ $\quad \cos A = \frac{4}{5}$

$\tan A = \frac{3/5}{4/5} = \frac{3}{4}$ $\quad \csc A = \frac{5}{3}$

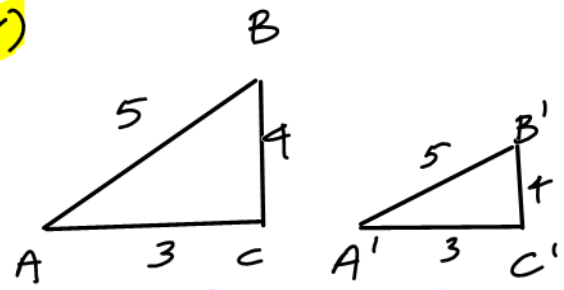
$\sec A = \frac{5}{4}$ $\quad \cot A = \frac{1}{\tan A} = \frac{4}{3}$

$\sin B = \frac{4}{5}$ $\quad \cos B = \frac{3}{5}$

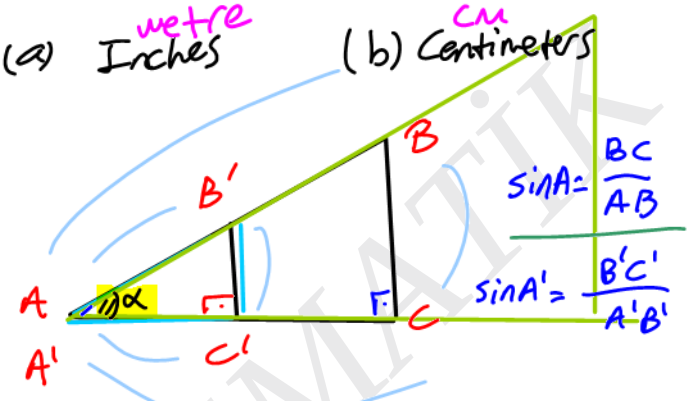
$\tan B = \frac{4}{3}$ $\quad \csc B = \frac{5}{4}$

$\sec B = \frac{5}{3}$ $\quad \cot B = \frac{3}{4}$

(**)



(a) Inches (metre) (b) Centimeters (cm)



$\Delta ABC \sim \Delta A'B'C'$

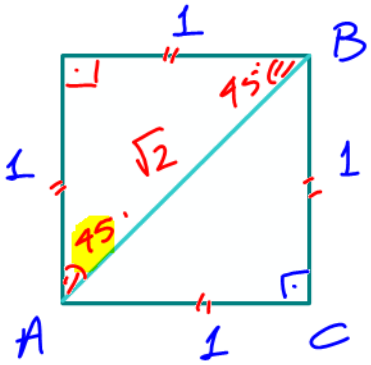
$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'} = k$

$\sin A = \frac{BC}{AB}$ $\quad \sin A' = \frac{B'C'}{A'B'}$

(*) $\sin A = \sin A'$

$\frac{\sin A}{\sin A'} = \frac{BC}{AB} \cdot \frac{A'B'}{B'C'} = k \cdot \frac{1}{k} = 1$

(*) $45°$'nin tüm değerlerini bulunuz.



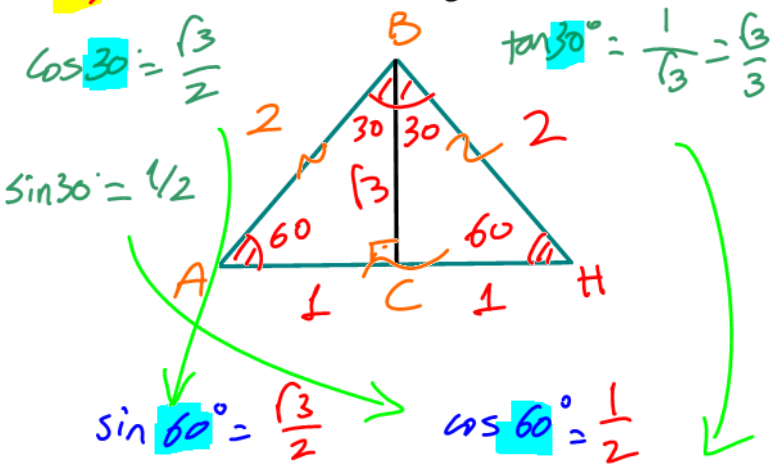
$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \cos 45° = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$ $\quad \cot 45° = 1$

$\csc 45° = \sqrt{2}$ $\quad \sec 45° = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

TUNÇ KURT MATEMATİK

(*) 60° 'nin tüm değerlerini bulunuz.



$\cos 30° = \frac{\sqrt{3}}{2}$ $\quad \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

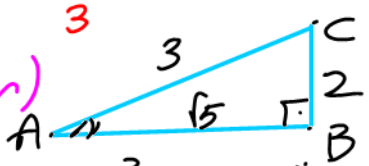
$\sin 30° = 1/2$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\quad \cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$ $\quad \cot 60° = \frac{\sqrt{3}}{3}$

$\csc 60° = \frac{2\sqrt{3}}{3}$ $\quad \sec 60° = 2$

(Dik üçgen)



Örnek: $\sin A = \frac{2}{3}$ olan üçgende;

$\sin A = \frac{2}{3}$ $\quad \cos A = \frac{\sqrt{5}}{3}$

$\tan A = \frac{2}{\sqrt{5}}$ $\quad \cot A = \frac{\sqrt{5}}{2}$

$\csc A = \frac{3}{2}$ $\quad \sec A = \frac{3}{\sqrt{5}}$

Kofonksiyon Teoremi:

30-60
60-30
45-45

$A + B = 90°$

$\sin 30 = \cos 60$
$\tan 30 = \cot 60$
$\sin 45 = \cos 45$
$1/\cos A = \frac{1}{\sin B} = \csc B$

$\sin A = \cos B$ $\quad \sec A = \csc B$ $\quad \tan A = \frac{\sin A}{\cos A} = \frac{\cos B}{\sin B} = \cot B$

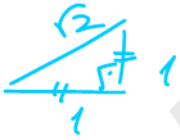
$\sin B = \cos A$ $\quad \sec B = \csc A$ $\quad \tan B = \frac{\sin B}{\cos B} = \frac{\cos A}{\sin A} = \cot A$

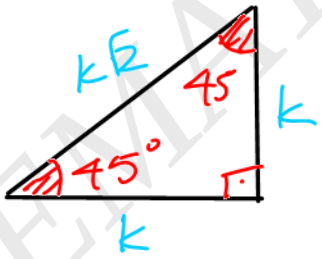
Örnek: $\sin 65° = \cos 25°$

$\cos 68° = \sin 22°$

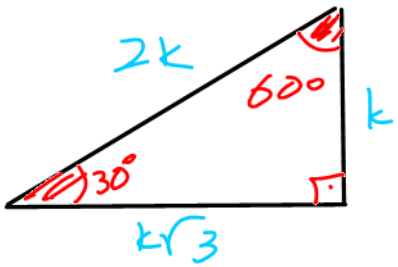
$\tan 59° = \cot 31°$

45-45-90:





30-60-90:

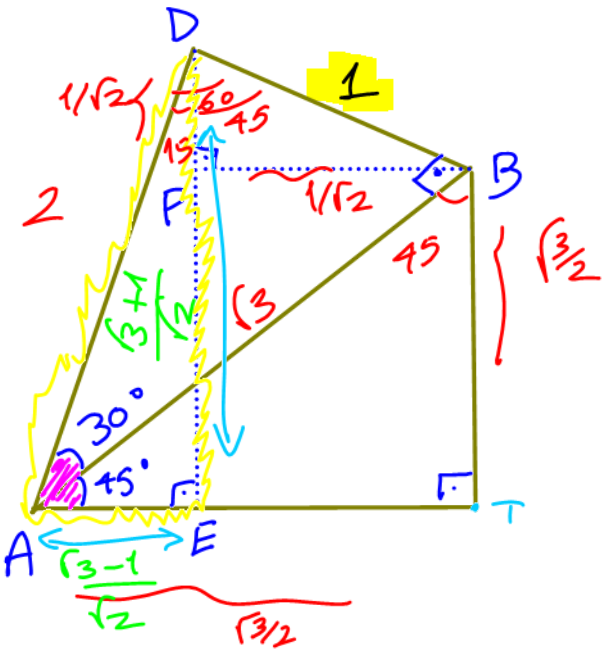


TUNÇ KURT MATEMATİK

$\sin(75) = \sin(45+30)$

$\sin(A+B)$

(*) 75°'nin tüm değerlerini bulunuz.

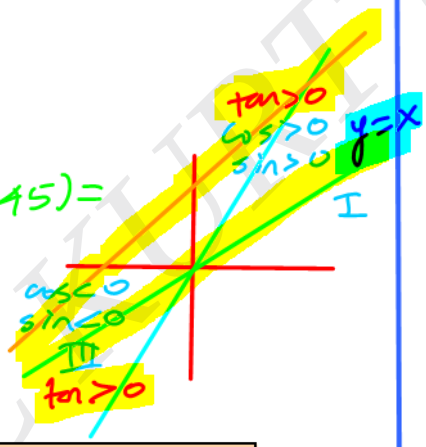


$AE = AT - ET = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}}$

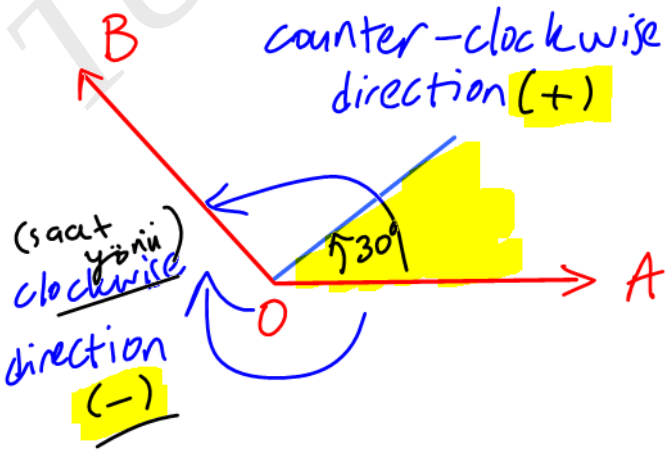
$DE = DF + FE = \frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{\sqrt{2}}$

$\sin 75^\circ = \ldots$

$\rightarrow \sin(30+45) =$



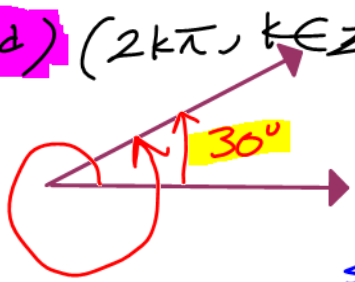
1.3. Herhangi Bir Açıda Trigonometrik Fonksiyonlar



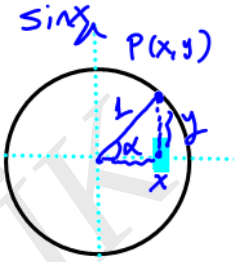
$360 \pmod{} \quad (2k\pi, k \in \mathbb{Z})$

$490 \mid 360$
$360 \mid 1$
130

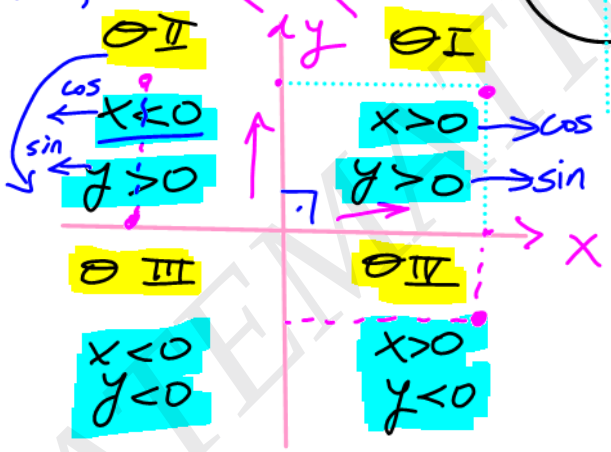
390°

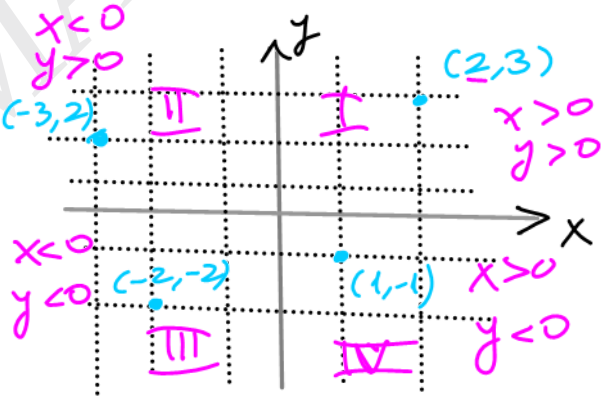


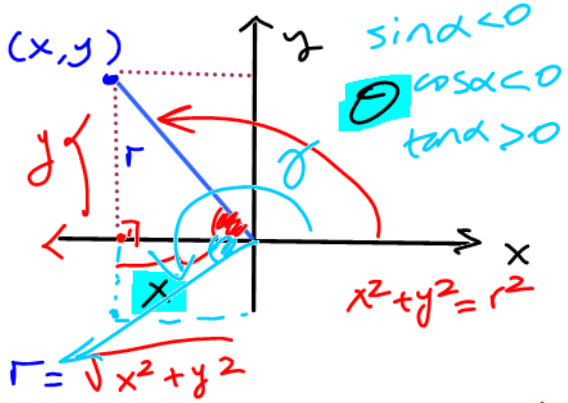
$390 = 360 + 30^\circ$



$\cos\alpha = x$
$\sin\alpha = y$
$x^2+y^2 = 1$





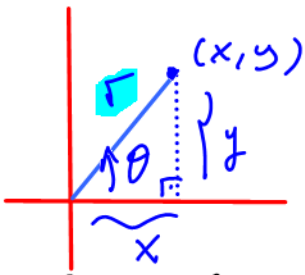


$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{-x}{r}$

$\tan\theta = -\frac{y}{x}$ $\quad \cot\theta = -\frac{x}{y}$

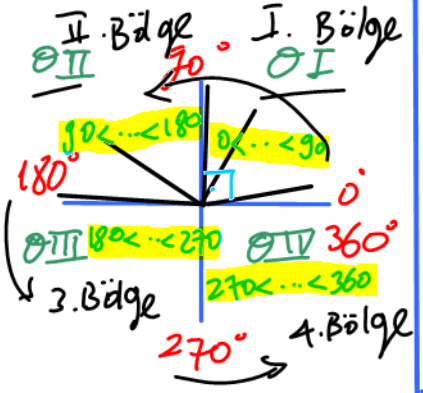
$\csc\theta = \frac{r}{y}$ $\quad \sec\theta = -\frac{r}{x}$

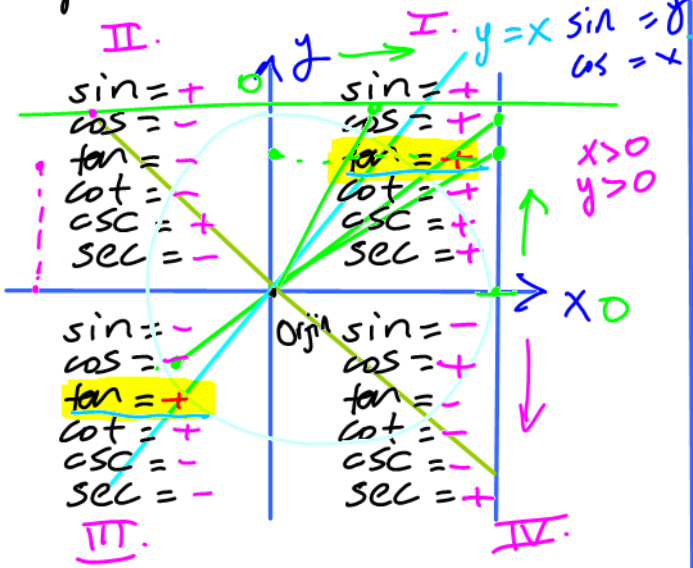
TUNCKURT MATEMATİK

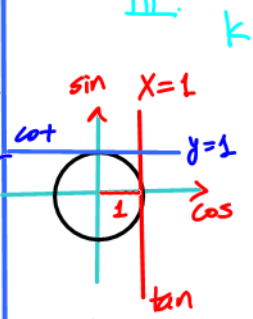


$x^2+y^2=r^2$

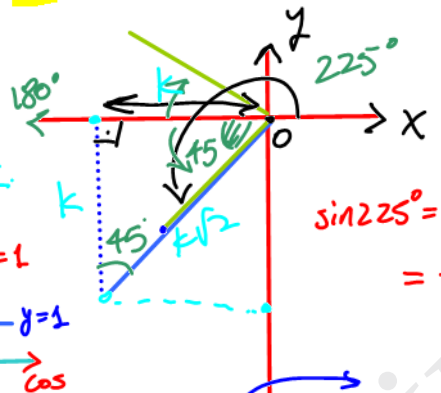
$\sqrt{x^2+y^2}=r$







Örnek (*) 225°'nin tüm değerlerini bulunuz



$\sin 225^\circ = \sin 45 = -\frac{1}{\sqrt{2}}$

$\sin(180+45) = -\sin 45 = -\frac{\sqrt{2}}{2}$

$\sin 225^\circ = -\frac{\sqrt{2}}{2}$ $\quad \cos 225^\circ = -\frac{\sqrt{2}}{2}$

$\tan 225^\circ = 1$ $\quad \cot 225^\circ = 1$

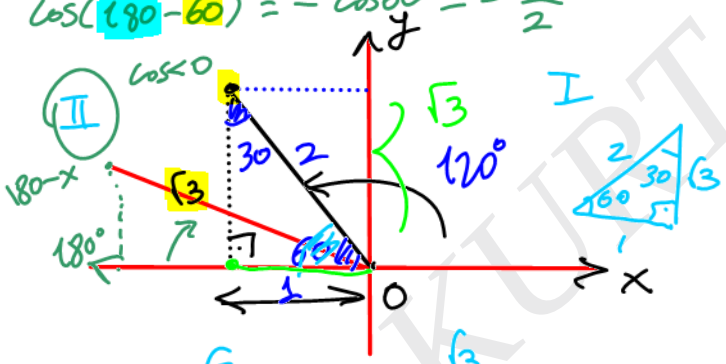
$\csc 225^\circ = -\frac{2}{\sqrt{2}}$ $\quad \sec 225^\circ = -\frac{2}{\sqrt{2}}$

$\tan(180+45) = \frac{\sin(180+45)}{\cos(180+45)} = \frac{-\sin 45}{-\cos 45} = \tan 45^\circ$

$0 < \alpha < 360$ (Esas ölçü)

Örnek (*) 120°'nin tüm değerlerini bulunuz

$\cos(180-60) = -\cos 60 = -\frac{1}{2}$



$\sin 120^\circ = \frac{\sqrt{3}}{2}$ $\quad \sin 60 = \frac{\sqrt{3}}{2}$

$\cos 120^\circ = -\frac{1}{2}$ $\quad \cos 60^\circ = \frac{1}{2}$

$\sin 120^\circ = \frac{\sqrt{3}}{2}$ $\quad \cos 120^\circ = -\frac{1}{2}$

$\tan(180-60) = -\tan 60 = -\frac{\sqrt{3}}{2}$

$\tan 120^\circ = -\frac{\sqrt{3}}{2}$ $\quad \cot 120^\circ = -\frac{2}{\sqrt{3}}$

$\sec 120^\circ = -2$ $\quad \csc 120^\circ = \frac{2}{\sqrt{3}}$

$\cos(180-x) = -\cos(x)$

$\sin(180-x) = +\sin(x)$

$\tan(180-x) = \frac{\sin(180-x)}{\cos(180-x)} = \frac{\sin x}{-\cos x} = -\tan x$

Örnek (*) 330°'nin tüm değerlerini bulunuz

$\sin(-30) = \sin(-30+360) = \sin(330) = -\sin 30$

$\cos(360-30) = +\cos 30 = \frac{\sqrt{3}}{2}$

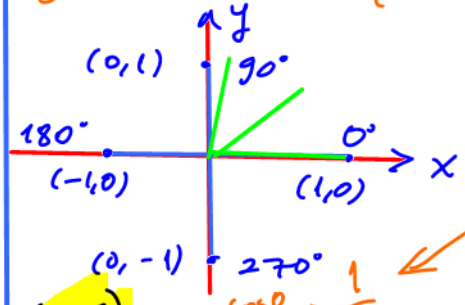
$\sin(360-30) = -\sin 30 = -\frac{1}{2}$

$\sin 330^\circ = -\frac{1}{2}$ $\quad \cos 330^\circ = \frac{\sqrt{3}}{2}$

$\tan 330^\circ = -\frac{1}{\sqrt{3}}$ $\quad \cot 330^\circ = -\sqrt{3}$

$\sec 330^\circ = \frac{2}{\sqrt{3}}$ $\quad \csc 330^\circ = -2$

$\frac{0}{0}$ = Belirsiz $= \frac{x-2}{1-1} = \frac{2(1-1)}{(1-1)} = 2$



$\sin 0^\circ = 0$

$\cos 0^\circ = 1$

$\tan 0^\circ = \frac{0}{1} = 0$

$\cot 0^\circ$ = Tanımsız

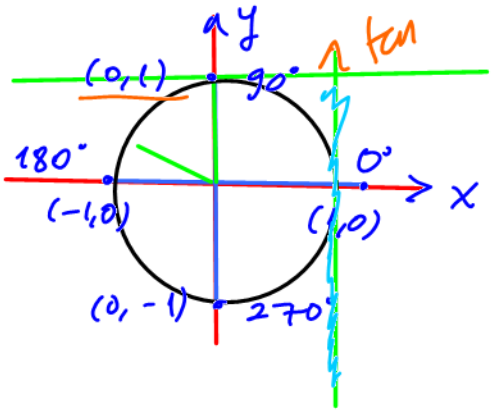
$\sec 0^\circ = 1$

$\csc 0^\circ$ = Tanımsız

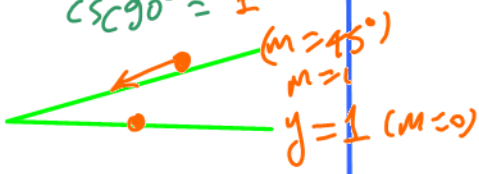
(*)

$\frac{\cos 0}{\sin 0} = \frac{1}{0}$ = Tanımsız

TUNÇ KURT MATEMATİK



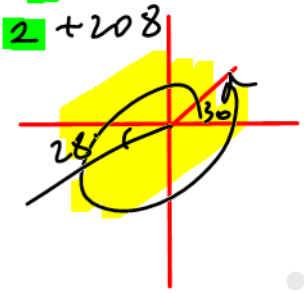
$\sin 90° = 1$
$\cos 90° = 0$
$\tan 90° =$ Tanımsız
$\cot 90° = 0$
$\sec 90° =$ Tanımsız
$\csc 90° = 1$



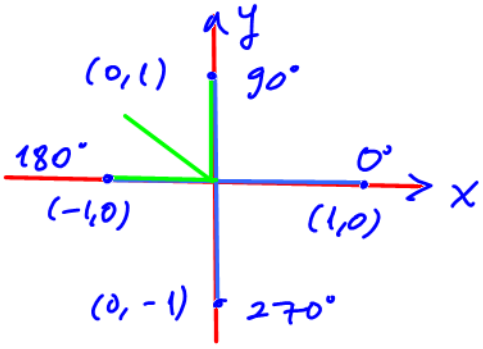
Örnek: 928° esas ölçüsünü bulalım.

$928 = 360 \cdot 2 + 208$

720
208



$0 < \alpha < 360$



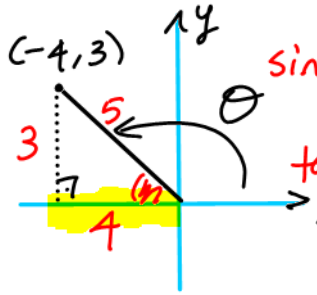
$\sin 180° = 0$
$\cos 180° = -1$
$\tan 180° = 0$
$\cot 180° =$ Tanımsız
$\sec 180° = -1$
$\csc 180° =$ Tanımsız

$0 < \alpha < \frac{\pi}{2}$

$\frac{\pi}{2} < \alpha < \pi$

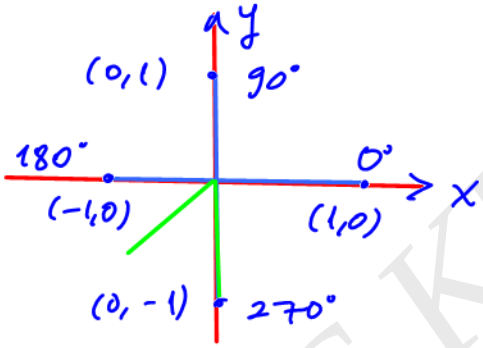
Örnek: $\cos\theta = -\frac{1}{5}$. $\sin\theta$ ve $\tan\theta$ bulalım.

II.



$\sin\theta = \frac{3}{5}$

$\tan\theta = -\frac{3}{4}$



$\sin 270° = -1$
$\cos 270° = 0$
$\tan 270° =$ Tanımsız
$\cot 270° = 0$
$\sec 270° =$ Tanımsız
$\csc 270° = -1$

$\sin\theta = -\frac{3}{5}$ $\quad \tan\theta = \frac{3}{4}$

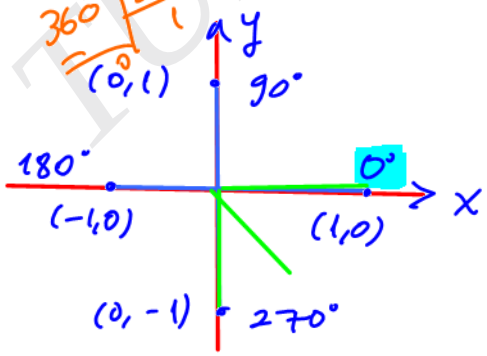
$372° = 12°$

$360 = \text{mod}$

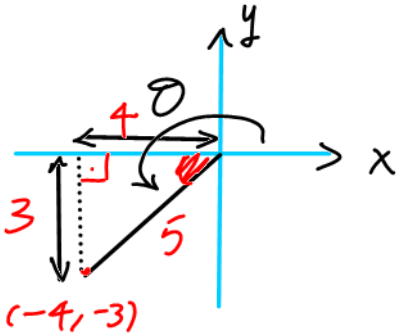
$360 \mid 360$
$0 \quad 1$

$\text{mod } 2 = \{0, 1\}$

$\hookrightarrow -1, 3, 99, -501$



$\sin 360° =$
$\cos 360° =$
$\tan 360° =$ (*)
$\cot 360° =$
$\sec 360° =$
$\csc 360° =$



III.

$0° = 360°$

TUNÇ KURT MATEMATİK

TUNÇ KURT MATEMATİK